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Sound Sources in Aerodynamics—Fact and Fiction

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Introduction

NOISE is distinguished from sound only in that it is an unwanted irritant. There is no measure of signal continuity or harmony that can classify a sound as noise. Its noisiness depends on the sensitivity of a listener and that sensitivity can vary with circumstances and time. The control of noise calls for definite measures and standards and those standards must reflect the magnitude of the noise nuisance. Since that is a subjective matter, it follows that the units of noise measurements are inevitably influenced by psychological issues. Because of this noise control is not as rational a subject as others in mechanics.

But the basic phenomenon of sound is entirely rational. Sound is a vibration of the air. It travels; an underwater pistol shot can be heard around the world and China's offshore islands hear clearly the news and comments shouted at them through powerful loudspeakers some 30 km away. Those sound waves carry tens of kilowatts of acoustical power, the equivalent of the world's noisiest aircraft, and when one recalls that the ear is accustomed to monitoring the human whisper, an acoustic power level of only 10^{-10} W, one can marvel at the amplitude range over which the sound is essentially linear. The ear detects pressure variations as small as 10^{-10} atm and can be damaged if subjected to variations in excess of 10^{-3} atm. Pressure fluctuations adjacent to the loudest jet aircraft reach a tenth of an atmosphere. Generally, therefore, sound pressure variations are weak, as are their associated aerial motions, and the mechanics of sound can be understood within the context of linear theory. In linear theory, waves add without distortion; multiple conversations at a cocktail party do not interfere.

Though linear sound is such a definite phenomenon, conforming precisely with the rational laws of mechanics, the source of that sound is not. That is an issue not dissimilar to the distinction between noise and sound; it is largely a point of view and subject to rational analysis only when some axiomatic position is taken. A source must first be defined. This point is true of all linear waves, be they optical, mechanical, or electrical, the issue being illustrated by considering the difficulty that an observer of a perfectly reconstructed holographic image would have in determining the source of the light field; and this example illustrates also that wave fields, identical in extensive regions of space, can have completely different sources. It is not possible to determine from a wave field what its source must have been. Of course, in linear fields one can make a good guess, holographic reconstructions are not very common. We rarely make mistakes in giving the most obvious interpretation to visual images. Blind bats are effective hunters of prey they track by interpreting the origin of scattered sound waves. The essential ambiguity in that interpretation bothers them not at all; but it becomes important once nonlinearities are admitted. Sources of sound in aerodynamics are frequently nonlinear and we shall see that determining whether certain features are, or are not, sources of sound, is sometimes influenced greatly by matters of formal definition. There is no doubt that if a flow is known precisely then its distant wave field is also known, but aerodynamicists are led to conjecture what aspects of a flow cause the noise field observed at a distance without a detailed knowledge of the flow; there is no unambiguous procedure by which that can be done.

Confidence in their conjectures must be established slowly by testing source location schemes in situations where the flow is understood and by examining details of sound production in simple flows that are known exactly. Unfortunately, not very many unsteady compressible flows are known exactly and much of recent progress in aerodynamic noise has concerned the development of asymptotic solutions to definite model problems; it is those solutions that give confidence in the source identifications made by formal but rather arbitrary prescriptions.

Not all that is heard is sound. The air stream about the unprotected ears of a motor cyclist can be painfully noisy even though a negligible part of the pressure variations within the turbulent eddies that bother him so much propagate away as sound. They can be made to propagate by the action, for example, of a resonator, and that is the principle of the wind instruments. Resonators and the way sound propagates from them have been understood as long as acoustics has been a science, but the mechanics of their excitation, i.e., the basic source process, is much more difficult to grasp. Some of that difficulty is caused by the pseudosound in which unsteady pressures are nonlinearly related to fluid motion and which are not organized like proper sound into propagating waves.

Sources of Sound

Because there is an essential ambiguity in the source of a wave it is convenient to take a formal position, recognizing in doing so that the view is only one of many possible ways of proceeding. Here we insist that the wave field does not

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overlap the source region in following the most commonly accepted definition of sound.

Sound is a linear motion in which all its elements, pressure, density, velocity, etc. satisfy the equation

$$\Box^2\phi=\frac{\partial^2\phi}{\partial t^2}-c^2\nabla^2\phi=0 \tag{1}$$

where c is the speed of sound; it is a constant in this linear field.

Now it is a fact that if Eq. (1) is valid throughout all of space, there can be no outgoing sound waves. $\phi=0$, i.e., absolute silence, is the only solution consistent with the radiation condition. There is no sound because there is no source. $\Box^2\phi$ is the source. We call it Q. Once Q is known, ϕ can be determined uniquely. In the sound field, by our definition of sound, Q is zero. The sources and sound do not overlap. That many source descriptions are possible is made obvious by recognizing that the sound field ϕ generated by the source field Q is identical to the sound field ϕ plus any function of Q that vanishes for zero Q.

$$\Box^2(\phi+f(Q))=Q+\Box^2 f(Q) \tag{2}$$

The source fields Q and $Q+\Box^2 f(Q)$ are different; their sounds are identical. It is clear from this that what is, or is not, the source of sound is largely a point of view.

However, all of space is never occupied by fluid and the preceding observation concerning an equation valid throughout all space is therefore not directly useful; we generalize it to apply to bounded volumes by the following trick. Define H to be the Heaviside function equal to unity in the fluid and zero elsewhere. The sound field ϕ in the fluid is therefore the same as the sound field $H\phi$. Also, the source field Q is there the same as the source field HQ. We can multiply Eq. (1), which is defined in the fluid by the Heaviside function H, to generate an equation valid throughout all space. It has the trivial value of zero outside the fluid. We then rearrange the equation into a wave equation for the sound field $H\phi$

$$H\Box^2\phi=HQ$$

$$H\left(\frac{\partial^2\phi}{\partial t^2}-c^2\nabla^2\phi\right)=\frac{\partial^2}{\partial t^2}HQ-c^2\nabla^2 H\phi+c^2\frac{\partial}{\partial x_i}\left(\phi\frac{\partial H}{\partial x_i}\right)+c^2\frac{\partial\phi}{\partial x_i}\frac{\partial H}{\partial x_i}$$

$$\Box^2 H\phi=HQ-c^2\frac{\partial}{\partial x_i}\left(\phi\frac{\partial H}{\partial x_i}\right)-c^2\frac{\partial\phi}{\partial x_i}\frac{\partial H}{\partial x_i} \tag{3}$$

The right-hand side of this equation is the source of the sound field $H\phi$ which, in the fluid where $H=1$, is identical to the sound field ϕ. We see that sound can be generated by two distinct source types. First, $\Box^2\phi=Q$ acts as a source distributed in the fluid; and, second, since $\partial H/\partial x_i$ is zero everywhere except on the surface which bounds the fluid, there are surface source distributions, a dipole of strength proportional to ϕ and a monopole of strength proportional to the gradient of ϕ.

Linear Noise Sources

The linearized equations of mass, continuity, and momentum conservation for an inviscid gas combine to give

$$\Box^2 p=\frac{\partial^2 p}{\partial t^2}-c^2\nabla^2 p=\frac{\partial^2 p}{\partial t^2}-c^2\frac{\partial^2\rho}{\partial t^2} \tag{4}$$

where p is the pressure fluctuation and ρ the mass density; ρ, but not p, may have strong gradients if this weakly disturbed gas is inhomogeneous.

Changes in the pressure of a fluid particle can be related to changes in its density once it is known how much heat is added to that particle. To do this we write $\rho=\rho(p,q)$ where q is the heat added per unit mass.

$$\Delta\rho=\left.\frac{\partial\rho}{\partial p}\right|_q\Delta p+\left.\frac{\partial\rho}{\partial q}\right|_p\Delta q \tag{5}$$

$$\left.\frac{\partial\rho}{\partial p}\right|_q=\frac{1}{c_l^2}$$

where c_l is the "local" speed of sound at that fluid particle. For a perfect gas $\rho=p/RT$ and $c_l^2=\gamma RT=c_p(\gamma-1)T$ so that

$$\left.\frac{\partial\rho}{\partial q}\right|_p=-\frac{p}{RT^2}\left.\frac{\partial T}{\partial q}\right|_p=-\frac{p}{RT^2c_p}=-\frac{\rho}{c_pT}=-\frac{(\gamma-1)\rho}{c_l^2} \tag{6}$$

and it then follows that the rate of change of the density of a fluid particle $D\rho/Dt=(\partial\rho/\partial t)+u_i(\partial\rho/\partial x_i)$ is

$$\frac{D\rho}{Dt}=\frac{1}{c_l^2}\frac{Dp}{Dt}-\frac{(\gamma-1)\rho}{c_l^2}\frac{Dq}{Dt} \tag{7}$$

The density gradient term is retained in the linear theory only to account for the possibility that the gas is inhomogeneous. This is made explicit if we write

$$\frac{\partial\rho}{\partial t}=\frac{D\rho}{Dt}-u_i\frac{\partial(\rho-\rho_0)}{\partial x_i}$$

ρ_0 being the uniform density of the fluid into which sound is radiating.

This equation can in turn be rewritten using the continuity equation

$$\frac{\partial\rho}{\partial t}=\frac{D\rho}{Dt}-\frac{\partial}{\partial x_i}\{(\rho-\rho_0)u_i\}+(\rho-\rho_0)\left\{\frac{\partial u_i}{\partial x_i}=-\frac{1}{\rho}\frac{D\rho}{Dt}\right\}$$

from which it follows that

$$\frac{\partial\rho}{\partial t}=\frac{\rho_0}{\rho}\frac{D\rho}{Dt}-\frac{\partial}{\partial x_i}\{(\rho-\rho_0)u_i\} \tag{8}$$

or, on making use of Eq. (7),

$$\frac{\partial\rho}{\partial t}=\frac{\rho_0}{\rho}\frac{1}{c_l^2}\frac{Dp}{Dt}-\frac{(\gamma-1)\rho_0}{c_l^2}\frac{Dq}{Dt}-\frac{\partial}{\partial x_i}\{(\rho-\rho_0)u_i\} \tag{9}$$

Since Dp/Dt is indistinguishable from $\partial p/\partial t$ on linear theory as is $\partial/\partial t\, Dq/Dt$ from D^2q/Dt^2, the right-hand side of Eq. (4) which are the linear sources of sound in an unbounded perfect gas, become

$$\Box^2 p=\left\{1-\frac{\rho_0c^2}{\rho c_l^2}\right\}\frac{\partial^2 p}{\partial t^2}+\frac{\rho_0c^2}{c_l^2}(\gamma-1)\frac{D^2q}{Dt^2}+\frac{c^2\partial}{\partial x_i}\left\{(\rho-\rho_0)\frac{\partial u_i}{\partial t}\right\} \tag{10}$$

In the perfect gas at constant pressure, $\rho_0c^2=\rho c_l^2$ so that the linear source terms simplify still further to give, finally,

$$\Box^2 p=(\gamma-1)\rho\frac{D^2q}{Dt^2}+\frac{c^2\partial}{\partial x_i}\left\{(\rho-\rho_0)\frac{\partial u_i}{\partial t}\right\} \tag{11}$$

Unsteady heat addition accounts for a monopole source of sound, and the acceleration of fluid particles of density different from their environment induces a dipole source of acoustic radiation. These are the only linear sources of sound in an unbounded perfect inviscid gas that is weakly disturbed from rest; the result holds true also for an inhomogeneous mixture of perfect gases providing they share a common ratio of specific heats.

Boundary effects are easily incorporated in the same way as they are in Eq. (3). The pressure replaces ϕ in that equation and $\partial p/\partial x_i$ is written as $-\rho\partial u_i/\partial t$ in the generalization of Eq. (11) to include surface sources.

$$\Box^2 Hp = H\left\{(\gamma-1)\rho\frac{D^2 q}{Dt^2}+\frac{c^2\partial}{\partial x_i}\left[(\rho-\rho_0)\frac{\partial u_i}{\partial t}\right]\right\}$$

$$-c^2\frac{\partial}{\partial x_i}\left\{p\frac{\partial H}{\partial x_i}\right\}+\rho c^2\frac{\partial u_i}{\partial t}\frac{\partial H}{\partial x_i} \qquad (12)$$

Boundary effects induce an additional dipole source whose strength density is set by the force exerted on the fluid by unit surface area. Vibration of the boundary induces a monopole source.

The effectiveness of these sources in generating sound depends on their distribution in space and in time, in particular it depends on whether the sources are large or small in comparison with the wavelength of sound they radiate.

Sound of Compact Sources

As a general rule, the most ineffective sources of sound are small ones. The measure of smallness is set by the acoustic wavelength, which is the only scale relevant to the acoustic field. If the source is compact, that is, it is small in comparison with the wavelength, it is of low acoustic efficiency. Conversely, the efficient acoustic sources are big and rapidly changing and generate sound of frequency high enough to have wavelength smaller than, or at least comparable to, the source dimension.

The sound of the source field

$$\Box^2 p = Q$$

is

$$p(x,t)=\frac{1}{4\pi c^2}\int Q\left(y,t-\frac{|x-y|}{c}\right)\frac{d^3y}{|x-y|} \qquad (13)$$

and, when the source is compact, the variation across the source of retarded time, $|x-y|/c$, needed to account for the acoustic delay between the emission of the sound at y and its reception at x, is negligible. So is the variation of $|x-y|^{-1}$ for the sound heard more than a wavelength away. The pressure at distance r radiated by the source distribution is therefore

$$p(r,t)=\frac{1}{4\pi c^2 r}\left[\int Q\,dV\right](t-r/c) \qquad (14)$$

and the acoustic power, which is $4\pi r^2\overline{p^2}/\rho c$, is $(1/4\pi\rho c^5)\overline{Q^2}$, where Q^2 is the mean square value of the volume integrated source strength.

For example, if the sound is generated by the unsteady addition of heat, at frequency ω, Q^2 can be obtained from Eq. (12) and the acoustical power estimated as

$$\frac{(\gamma-1)^2}{4\pi}\frac{\omega^2}{c^2}\frac{P^2}{\rho c^3} \qquad (15)$$

P^2 is the mean square of the rate of heat addition occurring at frequency ω.

One kW of power added to the air as heat at a frequency of 1 kHz generates about 10^{-1} W of acoustical power. Tens of kilowatts of unsteady heat addition at the same frequency would generate a few hundred watts of acoustical power; this source mechanism might be a significant element in the noise of powerful furnaces and it is probably a prime source of internal noise in the gas turbine engine.

Most sources are less effective than such monopoles because the source distribution is arranged in a destructively interfering pattern. The dipole distribution has a source structure

$$\Box^2 p=\frac{\partial F_i}{\partial x_i} \qquad (16)$$

and the integrated source strength vanishes.

$$\int\frac{\partial F_i}{\partial x_i}dV=\int F_n ds$$

F_n is zero on this surface s that bounds the source. The dipole source generates sound only because of the small influence of retarded time variations across the compact source distribution, the distant dipole induced pressure field of Eq. (16) being

$$p(x,t)=\frac{1}{4\pi c^2}\frac{\partial}{\partial x_i}\int F_i(y,t-|x-y|/c)\frac{d^3y}{|x-y|}$$

$$=\frac{\cos\theta}{4\pi c^3 r}\frac{\partial}{\partial t}\left[\int F dV\right](t-r/c) \qquad (17)$$

where θ is the angle which the sound ray traveling to the distant observer at x makes with the dipole axis. The dipole induced sound power of a compact dipole of total mean square strength $\mathfrak{F}^2$

$$\mathfrak{F}^2=\overline{[\int F dV]^2}$$

is

$$\frac{\omega^2\mathfrak{F}^2}{12\pi\rho c^7} \qquad (18)$$

The compact dipole field associated with the acceleration of light gas particles is given in Eq. (11) from which $\mathfrak{F}$ can be evaluated as

$$\mathfrak{F}=c^2\Delta\rho\omega U l^3$$

and the associated acoustic power as

$$\frac{1}{12\pi}\frac{\omega^4 l^4}{c^4}\frac{(\Delta\rho)^2}{\rho^2}\frac{U^2}{c^2}\rho c^3 l^2$$

$\Delta\rho$, ω, U, and l signify, respectively, the strength of the density inhomogeneity, the frequency, the velocity level, and source scale. If, for example, the source of sound were a candle flame flickering at $\omega/2\pi=100$ Hz, the gas speed U being a meter/second, the density ratio $\Delta\rho/\rho$ one half and l 10^{-2} m, the acoustic power of the flickering flame works out at nearly 10^{-10} W, the same as the barely audible human whisper. This source mechanism has relevance to the powerful noise made by the mixing of hot jets as they entrain their cool environment, but there the motions involved are hardly small enough to fall within this linear theory.

Most linear sources of noise are caused by vibration and stresses on the boundary of the acoustic field. If that boundary is due to compact objects then the most efficient source will be the monopole which will generate an acoustic power $Q^2/4\pi\rho c^5$ where

$$Q=\int_{\text{volume}}\rho c^2\frac{\partial u_i}{\partial t}\frac{\partial H}{\partial x_i}dV=\int_{\text{surface}}l_i\rho c^2\frac{\partial u_i}{\partial t}ds \qquad (19)$$

l_i is the direction cosine of the surface normal leading into the fluid and out of the surface. A volume outflow from surface pulsations is needed to generate a compact monopole. For example, if the surface of the pulsating body of surface area l^2 vibrates with surface acceleration a

$$\overline{Q^2} = l^4 \rho^2 c^4 \overline{a^2}$$

and this generates an acoustic power $l^4 \rho \overline{a^2}/4\pi c$.

If the surface area is 10^{-2} m and the acceleration level of the surface vibration 10^3 g then the radiated power is a couple of watts and the maximum sound pressure in the vicinity of the source is some 140 dB, the level at which the human ear begins to suffer immediate and permanent damage.

An everyday example of such loud monopoles is provided by the pulsating exhaust of a high performance motor cycle. One liter exhausted 10,000 times per minute generates 140 dB at a distance of 10 m.

However, bodies often vibrate in a mode that conserves their displaced volume and for those the monopole strength is zero. The dipole element in Eq. (12) and the variation in retarded time across the monopole distribution are then both important. Consider specifically a compact rigid body vibrating without change of shape. Then the velocity over its interior is defined and is solenoidal so that the linear monopole term in Eq. (12) becomes

$$\rho c^2 \frac{\partial u_i}{\partial t}\frac{\partial H}{\partial x_i} = \rho c^2 \frac{\partial u_i}{\partial t}\frac{\partial}{\partial x_i}(H-1) = c^2 \frac{\partial}{\partial x_i}\left\{\rho \frac{\partial u_i}{\partial t}(H-1)\right\} \tag{20}$$

i.e., it degenerates into a dipole. The total linear dipole strength F_i is then

$$F_i = c^2 \int_{\text{volume}} \left\{\rho \frac{\partial u_i}{\partial t}(H-1) - p\frac{\partial H}{\partial x_i}\right\} \mathrm{d}V$$

$$= -c^2 \rho V \frac{\partial u_i}{\partial t} - c^2 \int l_i p \,\mathrm{d}s$$

$$= c^2 \left\{ f_i - \rho V \frac{\partial u_i}{\partial t} \right\} \tag{21}$$

where f_i is the force exerted on the body by the fluid and $-\rho V(\partial u_i/\partial t)$ the force required to overcome the inertia of the gas displaced by the foreign body of volume V. If the actual force equals this force then the body does not support a dipole source of sound; the dipole is due only to the excess of the body force over the unsteady displaced inertia term. A similarity can be seen between this term and the distributed dipole source that would be present if inhomogeneous gas is in unsteady motion. Equation (11) shows this term also to be the difference between the force accelerating the gas of local density ρ and that needed to accelerate gas of the ambient density ρ_0, i.e., the force needed to accelerate the gas of density ρ_0 that has been displaced by that of density ρ.

Body forces are commonplace sources of sound. The unsteady motion of wind through trees generates sound in this way and the propellers of ships and aircraft support these sources also. The unsteady lift on a bumble bee with wings flapping at 100 Hz might amount to 10^{-2} N in which case the nearby unsteady pressure generated by that lift is about 80 dB.

All these compact source processes generate sound as a weak by-product of their motion. Their acoustic efficiency is low and the more complex the source, the lower is that efficiency on account of destructive interference between the wavelets of the individual source elements. That interference is not destructive if the source is big.

Noncompact Sources

Acoustically efficient sources have to be noncompact, i.e., not small on the wavelength scale. For them the acoustic field is a major constraint on their dynamics. Acoustic loads are high on noncompact sources and it is, for this reason, often very difficult to determine the source level without also knowing the sound field.

Consider, by way of example, the sound generated by heat added uniformly over the interior of a spherical volume of radius a in an otherwise still and uniform fluid. Consider the extreme case when that heat is added impulsively and the linear source term of Eq. (11) is

$$\square^2 p = (\gamma - 1)\rho \frac{\partial^2 q}{\partial t^2} \tag{22}$$

where $\rho \partial q/\partial t$, the rate of heat addition per unit volume, is

$$\rho \frac{\partial q}{\partial t}(x,t) = QH(a - |x|)\delta(t) \tag{23}$$

The distant sound field generated by this impulsive heating is

$$\frac{p(r,t)}{(\gamma-1)} = \frac{1}{4\pi c^2 r}\frac{\partial}{\partial t}\int \rho \frac{\partial q}{\partial t}(y, t - |x-y|/c)\mathrm{d}^3 y$$

$$= \frac{1}{4\pi c^2 r}\frac{\partial}{\partial t}\int_{y_r = \text{const}} \int_{Y_r} \rho \frac{\partial q}{\partial t}\left(t - \frac{r}{c} + \frac{y_r}{c}\right)\mathrm{d}y_r \mathrm{d}s$$

$$= \frac{Q}{4\pi c^2 r}\frac{\partial}{\partial t}\int_{y_r = \text{const}} H(a - |y|)\delta\left(t - \frac{r}{c} + \frac{y_r}{c}\right)\mathrm{d}y_r \mathrm{d}s$$

$$= \frac{Q}{4\pi c r}\frac{\partial}{\partial t}\int_{y_r = r - ct} H(a - |y|)\mathrm{d}^2 s$$

$$= \frac{Q}{4cr}\frac{\partial}{\partial t}\{(a^2 - (r-ct)^2)H(a^2 - (r-ct)^2)\}$$

i.e.,

$$p(r,t) = \frac{(\gamma-1)Q}{2}\frac{(r-ct)}{r}H(a^2 - (r-ct)^2) \tag{24}$$

This wave is first heard at r at time $(r-a)/c$ and then has an amplitude $(\gamma-1)Qa/2r$. This amplitude falls linearly with time until the wave at r ceases abruptly, at time $(r+a)/c$, at which time the pressure is $-(\gamma-1)Qa/2r$. The wave varies on the time scale a/c when excited by a noncompact source, i.e., its frequency is set by the source size. This is in sharp contrast to the compact case where the wave scale is determined by the source frequency.

The energy radiated into the acoustic field is

$$4\pi r^2 \int \frac{p^2(r,t)}{\rho c}\mathrm{d}t = \frac{(\gamma-1)^2 Q^2 \pi a^2}{\rho c}\int_0^{2a/c}\left(1 - \frac{c\tau}{a}\right)^2 \mathrm{d}\tau$$

$$= \frac{2\pi a^3 (\gamma-1)^2 Q^2}{3\rho c^2} \tag{25}$$

The energy deposited in the gas during this impulsive heating is the value of Eq. (23) integrated over all space and time, i.e., $(4/3)\pi a^3 Q$. The efficiency of conversion of that heat into sound is, in this noncompact case, equal to $[(\gamma-1)^2 Q/2\rho c^2]$. This can be contrasted with Eq. (15), where heat is being added at a rate $P[=(4\pi/3)\omega a^3 Q]$ in a compact source and only a fraction $(\omega^3 a^3/c^3)[(\gamma-1)^2 Q/3\rho c^2]$ of that energy radiates into sound. The compact heat source is acoustically less efficient than the noncompact source by the cube of the (then small) compactness ratio, $\omega a/c$.

Another example of a noncompact source that demonstrates its high acoustic efficiency is provided by the noise made by moving bodies that are impulsively brought to rest, the slamming of a door, for example. Prior to the impulsive arrest, the air moves around the door and the motion of that air has kinetic energy. That energy remains in the air over the

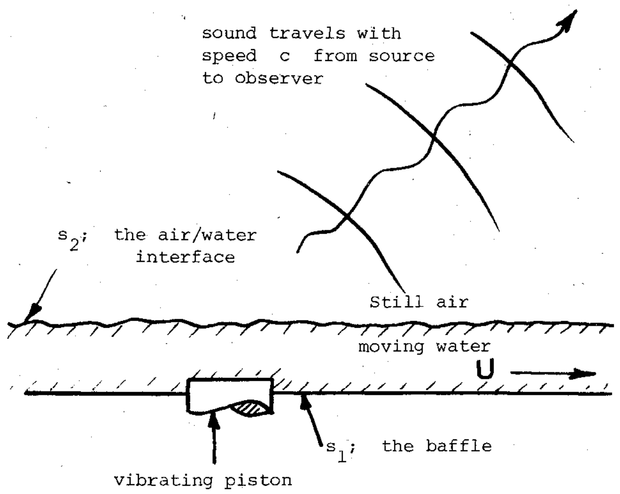

Fig. 1 Two views of the source process.

instant of impulsive arrest because the information that the door has stopped travels into the fluid at the finite speed of sound. None of the fluid is immediately aware of an impulsive boundary change. Since the energy of the moving fluid cannot be altered immediately by impulsive motion, and since it cannot be absorbed by the subsequently motionless boundary, it must go elsewhere; there are two possibilities. It could go into turbulence, to be dissipated locally by viscous action, or it could go into sound. The acoustic route is the more effective since the conversion of the local kinetic energy into sound is accomplished in the short time taken for sound to cross the disturbed volume of fluid. The viscous effects are smaller by a factor of the Reynolds number based on the speed of sound and the dimension of the body; a very large factor for a slamming domestic door. The air following a door slammed at 1 m/s is brought to rest by, and gives up its energy to, the 140 dB sound wave that is initiated at the impulse.

Acoustic Coincidence

Of course, many of the most important source fields fall between the compact and noncompact range. They have to be analyzed more formally in a procedure that accounts for the detailed balance between space and time scales, their ratio being a characteristic measure of a source speed. The ability of a source to radiate sound effectively depends on how large that speed is in comparison with the speed of sound. Slow motions are quiet, fast ones noisy.

Surface vibrations are treated usefully as waves, frequently they are bending waves, that travel in the surface at their own characteristic speed. Surface waves are coupled to the adjoining fluid which must vibrate in sympathy. A subsonically traveling surface wave supports a fluid wave that is attached to the surface. It carries no energy from the surface into the fluid and conveys none of the surface motion as sound to the distant field. Subsonic waves are compact and silent; but supersonically moving surface waves induce a sound field that must be supplied with energy to be radiated into the distant field. That energy drain damps out the surface wave which is intimately coupled to the attached sound field. The velocity at which the surface wave propagates is coincident with that of the oblique sound field that it launches. Coincident waves are damped by sound and the back reaction of the fluid on the surface vibration then can be highly significant.

Surface vibration generates sound only because of the supersonic wave elements in the vibration field. On a homogeneous surface those elements arise because surface waves move with supersonic phase velocity, but in an inhomogeneous surface the supersonic elements often arise because a continuous wave number spectrum is necessary to represent boundary constraints. The surface wave motion then may be mainly at subsonic phase speed but the spectrum is broadened by the constraints to include sound producing supersonic elements; stiffener and support elements can scatter into sound the energy of otherwise silent surface motions.

Nonlinear Sources

Having defined the source of sound according to Eq. (1), it is a relatively straightforward matter to evaluate it exactly. There are many possible descriptions of the source, depending on which variable one chooses to emphasize. By far the most compact form is due to Lighthill[1] who pioneered this style of analysis. He defined the source field to be $\square^2\rho$, where ρ is the mass density, and rearranged the exact statements of mass and momentum conservation as the inhomogeneous wave equation

$$\square^2\rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{26}$$

The linear sound field described by the left-hand side of this equation is driven by the nonlinear terms on the right-hand side; but there will be some linear terms on the right-hand side also, because the equation describes the laws of mechanics exactly and most linear motions are not sound waves. The presence of extensive "linear source terms" actually destroys the acoustic analogy which seeks to represent all motions as a superposition of appropriately forced sound waves; extensive linear source terms should be transferred to the left-hand side thus changing the wave operator and generating a new and different analogy in which the linear (and perhaps not in the least bit sound-like) field is driven by the nonlinear sources. Compact linear source regions are not a difficulty because they are so embedded in and constrained by their environment that it is reasonable to regard their values as being specified independently of the sound field. The sound of compact combustion, for example, is one that is generated by an effectively constant pressure source because the environment of a compact nonsingular source cannot sustain a significant pressure to react against and support a pressure buildup in the source. On the other hand, extensive distributions of heat can cause pressurization of the source, and it is then not reasonable to assume that that pressure and the nonlinear sources are known independently of sound; the pressure is part of the sound field which has to be solved.

As an example of the distorting influence of linear source terms, consider the acoustic analogy applied to the problem of calculating the sound radiated into air by the weak vibration of a piston baffled by a rigid horizontal plane surface that is covered with a thin layer of moving water, Fig. 1.

The exact solution of Lighthill's equation gives the density field at (x,t) as

$$\begin{aligned}\rho(x,t) &= \frac{1}{4\pi c^2}\frac{\partial^2}{\partial x_i \partial x_j}\int_v \left[\frac{T_{ij}}{r}\right] d^3y \\ &- \frac{1}{4\pi c^2}\frac{\partial}{\partial x_i}\int_s \frac{l_j}{r}[p_{ij} + \rho u_i u_j] d^2y \\ &+ \frac{1}{4\pi c^2}\int_s l_i \left[\frac{\partial \rho u_i}{\partial t}\right]\frac{d^2y}{r}\end{aligned} \tag{27}$$

The integration is over the volume v bounded by the surface s, which at first sight is obviously to be positioned on the baffle surface s_1. However, that positioning gives a formula for the field that, if casually interpreted with the superior performance of compact elementary sources borne in mind, is most misleading. The third term in the equation gives the monopole strength to be the rate of change in the mass flux at the piston face, a large term because the piston face is immersed in water and the appropriate value of ρ on that surface is the density of water. Also, there is an apparently powerful dipole with axis parallel to the water flow velocity, the linear term $l_i\rho Uu_i$ again being proportional to the mean momentum density of the water flow that grazes the piston surface. An alternative positioning of the control surface s at the air water interface s_2 reveals at once that those large source terms are spurious, for in this second positioning there is no mean velocity parallel to the surface and the fluid density there is the much smaller density of air. Neither is there any distributed quadrupole when this choice is made. The quadrupoles in the previous formula, because they contain linear terms are very far from negligible; they account precisely for the large difference between the two sets of surface sources at s_1 and s_2. It is the density term in T_{ij} that is the problem because in the water this term is so much larger than the pressure term which it balances in air. That term, when integrated over the water volume V_ω, gives a far acoustic field of

$$\frac{1}{4\pi c^2}\frac{\partial^2}{\partial x_i \partial x_j}\int_{V_\omega}\left[\frac{T_{ij}}{r}\right]\mathrm{d}^3y=\frac{1}{4\pi c^4}\frac{\hat{x}_i\hat{x}_j}{r}\frac{\partial}{\partial t}$$

$$\times\int_{V_\omega}\left[\frac{\partial}{\partial t}(c^2\rho\delta_{ij})\right]\mathrm{d}^3y=\frac{-1}{4\pi c^2 r}\frac{\partial}{\partial t}\int_{V_\omega}\left[\frac{\partial\rho u_i}{\partial y_i}\right]\mathrm{d}^3y$$

$$=-\frac{1}{4\pi c^2 r}\frac{\partial}{\partial x_i}\int_{V_\omega}\left[\frac{\partial\rho u_i}{\partial t}\right]\mathrm{d}^3y-\frac{1}{4\pi c^2 r}\int_{s_1+s_2} l_i\left[\frac{\partial\rho u_i}{\partial t}\right]\mathrm{d}^2y \tag{28}$$

This linear element of the quadrupole term therefore generates a field that exactly balances that apparently induced by the mass flux at the piston surface and also makes clear that it is only the imbalance of the unsteady drag force on the piston over the unsteady streamwise momentum in the water that gives rise to sound in the air; this imbalance is extremely small. Linear source terms must be avoided in clear expressions of the sound field.

The use of Lagrangian source coordinates avoids most of this type of ambiguity because the rates of change following a particle have none of the spuriously high valued terms arising from the silent convection of sharp gradients past a fixed point. Ffowcs Williams and Hawkings[2] expressed the exact solution to Lighthill's equation as

$$\rho(x,t)-\rho_0=\frac{1}{4\pi c^2}\frac{\partial^2}{\partial x_i\partial x_j}\int_v\left[\frac{T_{ij}J}{r|1-M_r|}\right]\mathrm{d}^3\eta$$

$$-\frac{1}{4\pi c^2}\frac{\partial}{\partial x_i}\int_s\left[\frac{l_i p_{ij}A}{r|1-M_r|}\right]\mathrm{d}^2\eta$$

$$+\frac{1}{4\pi c^2}\frac{\partial}{\partial t}\int_s\left[\frac{l_i\rho_0 u_i A}{r|1-M_r|}\right]\mathrm{d}^2\eta \tag{29}$$

with cM_r being the speed at which the particle at η is approaching x and J and A, respectively, the fractions by which the particle volumes and areas have changed since the time when the fluid particles were identified by their position $\eta=y$. This expression, unlike Eq. (27), gives an unambiguous statement of the sound field in the "flooded piston" problem regardless of whether s is made coincident with s_1 or s_2.

Unfortunately, flows for which the field integrals in either Eq. (27) or (29) are so easily interpretable are very rare and the best documented of those cases are ones in which small regions of homogeneous fluid contain low Mach number turbulent motion; they have relevance to the noise produced by subsonic air jets. It is then that the Reynolds stress elements of Lighthill's stress tensor generates the sound and Lighthill's formula for that sound,

$$\rho(x,t)=\frac{1}{4\pi c^4}\frac{x_ix_j}{x^3}\frac{\partial}{\partial t^2}\int_v[\rho u_iu_j]\mathrm{d}^3y \tag{30}$$

can be approximated at low enough Mach number into several equivalent forms, the most straight forward being

$$\frac{\rho}{\rho_0}(x_1=x,0,0,t)=\frac{1}{4\pi c^4 x}\frac{\partial^2}{\partial t^2}\int_v[u_1^2]\mathrm{d}^3y \tag{31}$$

Variations in the density form a negligible part of the stress tensor. For convenience the sound is observed at the distant point $x=(x,0,0)$ and it is only those velocity fluctuations in that direction that feature in the sound producing stress distribution.

Lighthill showed that at low Mach number the sources

$$\frac{\partial}{\partial t}u_iu_j \text{ and } pe_{ij}=p\left\{\frac{\partial u_i}{\partial y_j}+\frac{\partial u_j}{\partial y_i}\right\} \tag{32}$$

are identical so that the sound source can alternatively be thought of as the product of the pressure fluctuation and the strain rate in the direction of radiation.

$$\rho(x,0,0,t)=\frac{1}{2\pi c^4}\frac{\partial}{\partial t}\int_v\left[p\frac{\partial u_1}{\partial y_1}\right]\mathrm{d}^3y \tag{33}$$

The source field alternatively can be represented as a dipole distribution, the dipole form of Lighthill's formula Eq. (31) being

$$\frac{\rho}{\rho_0}(x,0,0,t)=\frac{1}{4\pi c^3 x}\frac{\partial}{\partial t}\int_v\left[\frac{\partial u_1u_i}{\partial y_i}\right]\mathrm{d}^3y \tag{34}$$

a form that was rewritten by Powell,[3] using the fact that the source velocity field could be regarded as solenoidal at low enough Mach number, to illustrate the importance of vorticity in the source process.

$$\frac{\rho}{\rho_0}(x,0,0,t)=\frac{1}{4\pi c^4}\frac{\partial^2}{\partial t^2}\int_v\left[\frac{1}{2}u^2\right]\mathrm{d}^3y$$

$$-\frac{1}{4\pi c^3 x}\frac{\partial}{\partial t}\int_v[u\wedge\omega]_1\mathrm{d}^3y \tag{35}$$

The first of these terms, representing as it does the kinetic energy of the turbulent fluid, varies at a negligible rate. Lighthill's source equivalence of Eq. (32) also shows that it is negligible. The second term would also vanish were it not for the influence of retarded time because the integral of $(u\wedge\omega)_1$ is exactly equal to the integral of

$$\frac{\partial}{\partial y_1}\left(\frac{1}{2}u^2\right)-\frac{\partial}{\partial y_i}(u_1u_i)$$

which must vanish if the source region is surrounded by linearly disturbed flow. The vortical source term must therefore be evaluated with care and this can be done by expanding $[u\wedge\omega]$ in a Taylor series to bring out the influence of retarded time variations. This leads to Powell's formula,

$$\frac{\rho}{\rho_0}(x,0,0,t)=\frac{1}{4\pi c^4 x}\frac{\partial^2}{\partial t^2}\int_v y_1(u\wedge\omega)_1\mathrm{d}^3y \tag{36}$$

The sound is here thought of as generated by the unsteady "vortex force" $\rho_0 u\wedge\omega$ per unit volume. Alternative views of the part played by vorticity are possible. Note, for example, that for a solenoidal velocity field,

$$y_1(u\wedge\omega)_1=\frac{1}{3}y\cdot(u\wedge\omega)+\frac{1}{3}\frac{\partial}{\partial t}\{y_1(y_2\omega_3-y_3\omega_2)\}$$

$$-\frac{1}{3}\frac{\partial}{\partial y_i}\{\omega_iy_1(y_2u_3-y_3u_2)-u_iy_1(y_2\omega_3-y_3\omega_2)\} \tag{37}$$

The last term, being a divergence, represents a source of higher order and negligible efficiency at low enough Mach number. The first term integrates to zero whenever the energy of the vortex field is conserved [c.f. Lamb,[4] Eq. (7), p. 218], as we have already assumed it to be in arriving at Eq. (36). The Powell expression for the sound field is therefore equivalent to that given by Möhring[5]

$$\frac{\rho}{\rho_0}(x,0,0,t) = \frac{1}{12\pi c^4 x}\frac{\partial^3}{\partial t^3}\int_v y_1(y_2\omega_3 - y_3\omega_2)\,\mathrm{d}^3y \tag{38}$$

a form that links the sound field "linearly" to the vorticity of the source flow. That linearity, however, is illusory because $\partial\omega/\partial t$ is an essentially nonlinear quantity being given by the Helmholtz vorticity equation

$$\frac{\partial\omega_j}{\partial t} = \frac{\partial}{\partial y_i}(\omega_i u_j - u_i\omega_j)$$

All these various representations of the nonlinear flow induced sources lead to a common prediction of an efficiency for the conversion of the flow's energy into sound that increases in proportion to the fifth power of Mach number; Lighthill's celebrated eighth power energy law. They also all point to the negligible effect of these sources in comparison with surface sources whenever the flow is adjacent to or disturbed by a foreign body, an airfoil supporting unsteady buffeting forces, for example. However, direct experimental checks on the validity of that view are often complicated by the subsequent interaction of sound with its parent flow or with the difficulty of conducting accurate and detailed experiment.

Consider, for example, what is at first sight one of the simplest test cases, that of a rigid airfoil undergoing small amplitude vibrations in a still fluid. The theory is unequivocal in its representation of the induced sound by a dipole distribution whose strength is determined by the airfoil shape, its velocity of oscillation, and the force it exerts on the fluid. Brooks[6] has measured these in considerable detail and failed to validate the theory which overpredicts the noise he actually observed. The cause of this overprediction has obviously become a matter of serious debate for it puts into doubt the very foundation of the subject. Yates[7,8] has drawn attention to the fact that the dissipation of energy by viscosity cannot be neglected in experiments of this type whenever the foreign bodies have sharp edges. No doubt that is true, but one would have hoped that viscous effects at airfoil trailing edges might have been accounted for, as they usually are in aerodynamics, by the imposition of some Kutta condition; viscosity causes the shedding of vorticity. Once that had been recognized and the influence of vorticity accounted for, the viscous effects would have been dealt with.

The fact that the energy dissipated by viscosity is of the same order as that radiated as sound is not in itself grounds for believing viscosity to have a direct bearing on the sound generation process. That is because the unsteady viscous traction forces on the surface of the airfoil do work and feature in the dipole terms of the exact acoustic analogy but were neglected in the Brooks experiment. Their work is likely to supply the energy demand of the dissipative viscous stresses in the fluid.

There are grounds for believing that the linear viscous terms are really very poorly coupled to the sound. Alblas,[9] for example, has demonstrated that they have a negligible influence on the diffraction of low-frequency sound waves by a sharp edge even though they dominate the flow in the vicinity of that edge. However, nonlinear vortical terms are important and it is known that vorticity shed from the trailing edge of a surface acts to inhibit the sound that would otherwise be radiated.[10] This may be a factor now in the Brooks experiment with nonlinear streaming convecting the vorticity downstream. Then there may have been more local nonlinear effects that went unnoticed in the experiment. There is a finite force exerted by the flow on a trailing edge. That force, which is known to be equivalent to Powell's vortex force,[11] is too concentrated to have been resolved by the sparse array of pressure transducers used by Brooks. Could this be the cause of the discrepancy? The resolution of that question is a task that could challenge experimentalists for some time to come.

It is very difficult to produce convincing arguments in problems like this from a purely theoretical treatment, mainly because the unresolved issues concern the often chaotic motion of inhomogeneous vortical flows. Sound sources have to be described in both space and time and we know so very little acoustically useful information on how turbulence evolves.

Eddies conforming with Taylor's hypothesis are silent so that the source activity in convected turbulence is concerned essentially with the evolution of eddies, an aspect that falls within the province of rapid distortion theory when the mean straining field is sufficiently strong. It is likely to be so in the most effective sound producing flows. That theory and a remarkable development of the source description due to Howe[12] shows that there is no upstream or downstream sound radiated by turbulence convected past and deformed by an obstacle in a stream. The sound is strongest in the cross-stream directions. This follows because Howe's analysis proves that low Mach number sound is generated in proportion to the rate at which vortex lines cross the streamlines of potential flow inclined at infinity in the direction of sound propagation. Rapid distortion theory involves vortex lines which follow the streamlines of the straining motion and which are therefore silent in the direction at which the stream approaches or leaves the distortion-producing body. This is an explanation of the frequently observed fact that unsteady lift forces are much stronger than the unsteady drag and so is the noise of the lifting forces, which radiate across the stream, much stronger than the streamwise drag-induced sound.

Progress in a subject as intricate as this, and one in which it is so difficult to produce accurate experimental data on well-defined flows, is very dependent on model problems which can be analyzed exactly. When those models contain vorticity concentrations their behavior frequently is puzzling and full of character. One of the most puzzling aspects of these models is described by Jones[13] who considered a line source adjacent to an initially plane vortex sheet. He showed how the early sound was reflected and transmitted through that sheet, i.e., how it interacted with the flow, in a perfectly definite manner, and how, after a certain time, the field became singular in some regions adjacent to the sheet. Jones brought in the powerful theory of ultradistributions to handle these unusual responses. The physical meaning to be attached to the ultradistributions was far from clear and it is of interest to examine the underlying cause of such effects that defy description in terms of more familiar functions. Those effects persist in the limit of zero Mach number and, therefore, can be found in the incompressible vortex sheet problem. That problem is quite straightforward.

Consider that an impulsive line source fires at time $t=0$ a distance d below a plane vortex sheet that lies at $y=0$. Above the sheet the fluid is initially at rest and subsequent motions are defined through a velocity potential ϕ^+ according to Laplace's equation

$$\nabla^2\phi^+ = 0; \quad y>0 \tag{39}$$

The incompressible fluid below the sheet is initially in uniform motion at speed U but is disturbed by the source by an amount described by the velocity potential ϕ^- which conforms with the Poisson equation

$$\nabla^2\phi^- = 2\pi\delta(x,y+d,t); \quad y<0 \tag{40}$$

The boundary conditions to be applied at the sheet are that the pressure should be continuous and that the sheet always forms an interface of contact between the upper and lower regions of fluid. Those conditions require that

$$\frac{\partial\phi^+}{\partial t}=\frac{D\phi^-}{Dt}$$

and

$$\frac{D}{Dt}\frac{\partial\phi^+}{\partial y}=\frac{\partial}{\partial t}\frac{\partial\phi^-}{\partial y}\quad\text{at}\quad y=0$$

with

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\tag{41}$$

The solution to these equations consistent with no disturbance at infinity gives

$$\phi^+=\delta(t)\ln\hat{r}_0+H(t)\frac{D}{Dt}\ln(rr_s)$$

and

$$\phi^-=\delta(t)\ln r_0+H(t)\frac{\partial}{\partial t}\ln(r'r_s')\tag{42}$$

where δ and H are, respectively, the delta and Heaviside functions and

$$r_0^2=(x/d)^2+(y/d+1)^2$$

$$\hat{r}_0^2=(x/d-Ut/d)^2+(y/d+1)^2$$

$$r^2=(x/d-\tfrac{1}{2}Ut/d)^2+(y/d+1-\tfrac{1}{2}Ut/d)^2$$

$$r'^2=(x/d-\tfrac{1}{2}Ut/d)^2+(1-y/d-\tfrac{1}{2}Ut/d)^2$$

$$r_s^2=(x/d-\tfrac{1}{2}Ut/d)^2+(1+y/d+\tfrac{1}{2}Ut/d)^2$$

$$r_s'^2=(x/d-\tfrac{1}{2}Ut/d)^2+(1-y/d+\tfrac{1}{2}Ut/d)^2\tag{43}$$

The flow induced by the source is as if there were singularities across the vortex sheet propagating at an angle of 45 deg, both their downstream and cross-stream propagation speeds being $\tfrac{1}{2}U$. The displacement of the vortex sheet ξ can be found easily by integration and, for $t>0$, is proportional to

$$\xi(x,t)=\frac{1}{2}\frac{1}{x^2/d^2+1}+\frac{x/d-\tfrac{1}{2}Ut/d}{(x/d-\tfrac{1}{2}Ut/d)^2+(1+\tfrac{1}{2}Ut/d)^2}-\frac{x/d-\tfrac{1}{2}Ut/d}{(x/d-\tfrac{1}{2}Ut/d)^2+(1-\tfrac{1}{2}Ut/d)^2}\tag{44}$$

The surface displacement becomes infinite at time $2d/U$ at the position $x=d$. Furthermore, at positions $x=d-\epsilon$ and $x=d+\epsilon$ it then has displacement proportional to $+\epsilon^{-1}$ and $-\epsilon^{-1}$; i.e., the vortex sheet has become discontinuous at this instant. It is obvious that the linear theory used in its study, which is the basis of the boundary conditions in Eq. (41) and Jones' treatment of the compressible problem, has become irrelevant. The vortex sheet response to an impulse is infinite at time $2d/U$ whatever the strength of the excitation. No wonder that strange functions are needed to represent its subsequent motion according to small perturbation theory! Vortex sheets, and indeed all unstable velocity profiles, play a far from obvious role in the theory of aerodynamic sound.

Some modeling of the primary flow is known to be needed to account for the interaction between sound and the various propagation and scattering effects that a flow can support. Because all interesting aeroacoustic problems concern turbulence, the chaotic consequence of a basic flow instability, the modelings that represent that flow must contain unstable solutions. Those models have either to be examined through a nonlinear phase into the turbulence consequential on the instability—a step that is hardly likely ever to prove practical—or some analogy is called for. The simplest analogies are linear and linear analogies are sensible only so long as the response of the system to excitation is at least bounded. Boundedness is incompatible with the linear causal response of an unstable system and insistence on boundedness inevitably makes the response anticipate its source. That source is turbulence, the production of which is associated with the noisy growth of instabilities and that noise necessarily anticipates the turbulence. Because what is or what is not a source is largely a point of view this violation of causality need cause no difficulty. The anticipatory response of the flow is an instability wave growing from zero to cancel that which will grow as a result of the source activity, and their mutual destruction ensures the necessary boundedness of the field. This view expanded by Dowling et al.[14] formalizes the role played by instabilities, but which was previously ignored, in other developments of Lighthill's analogy to include refraction effects. This view drives one to recognize also that the various turbulence source terms of unstable flows must be influenced by external sources, because the sum of the two noncausal responses must be causal to external effects. That fact was used by Ffowcs Williams and Purshouse[15] to examine the interaction that must exist between turbulence and a compliant boundary, and some of their deductions seem to have experimental verification. This kind of development is at an extremely exciting phase and may point eventually to exploitable effects of a quite unexpected kind; but they may point also to essential limitations of this style of analysis.

Conclusion

The difficult task of estimating the sources of noise at high Mach number, be it in free turbulent shear flows on, or near, the surfaces of transonic machinery, continues to challenge both the experimenter and theoretician alike. Different approaches suggest different prescriptions for those sources. It is those areas that have most bearing on today's aeronautical noise issues, issues that are, perhaps for a while, likely to take second place to ones of cost and fuel efficiency; but they are sure to regain prominence as today's new quiet aircraft are stretched and tuned to grow into the variants that the airlines will undoubtedly call for. It is probable that those variants will be noisy and that the noise problem will return to be again a critical constraint on their development.

Today's most pressing aeroacoustics problems are of a different kind and the most intense research effort is probably concentrated on the control of underwater noise, an area that permits the exploitation of the small Mach number parameter and makes relevant the various low Mach number source descriptions described earlier. The nonacoustic fall-out of the subject is also gathering pace. Aeroacoustics concerns the prediction and understanding of weak fields induced as a by-product of inhomogeneous flows, the secondary effects of turbulence, and, most recently, with the back-reaction of those fields on the turbulence. That is exactly the same area as aerodynamic instability, flutter, the response of airfoil cascades, compressors, and turbines, distorted inflow, and possibly the partial control of turbulence by external stimuli. All of these areas are of very great practical concern and, together with the aircraft noise question, will provide the stimulus needed for the continued and vigorous development of the subject.

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